

# Quasi-two-body decays $B_{(s)} \rightarrow P\rho \rightarrow P\pi\pi$ in perturbative QCD approach

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In this work, we calculate the  $CP$ -averaged branching ratios and the direct  $CP$ -violating asymmetries of the quasi-two-body decays  $B_{(s)} \rightarrow P(\rho \rightarrow)\pi\pi$  by employing the perturbative QCD (PQCD) approach (here  $P$  stands for a light pseudoscalar meson  $\pi, K, \eta$  or  $\eta'$ ). The vector current time-like form factor  $F_\pi$ , which contains the final state interactions between the pion pair in the resonant region associated with the  $P$ -wave states  $\rho(770)$  along with the two-pion distribution amplitudes, are employed to describe the interactions between the  $\rho$  and the pion pair under the hypothesis of the conserved vector current. We found that (a) the PQCD predictions for the branching ratios and the direct  $CP$ -violating asymmetries for most considered  $B_{(s)} \rightarrow P(\rho \rightarrow)\pi\pi$  decays agree with currently available data within errors; (b) for  $B(B \rightarrow \pi^0 \rho^0 \rightarrow \pi^0(\pi^+\pi^-))$ , the PQCD prediction is much smaller than the measured one; and (c) for  $B^+ \rightarrow \pi^+(\rho^0 \rightarrow)\pi^+\pi^-$  decay mode, we found a negative  $CP$  asymmetry  $(-27.5_{-3.7}^{+3.0})\%$ , which agrees with other theoretical predictions but different in sign from those as reported by BaBar and LHCb Collaboration.

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## I. INTRODUCTION

Experimental data from different collaborations, like BaBar [1–5], Belle [6–9] and LHCb [10–12], provide valuable information for the three-body hadronic  $B$  meson decays. For these decay modes, both the resonant and nonresonant contributions may appear, as well as the possible significant final state interactions (FSIs) [13–15]. Different frameworks have been developed for the study of the three-body hadronic  $B$  meson decays, based on the symmetry principles [16–24] or factorization theorems [25–34]. The QCD-improved factorization (QCDF) [31–34] has been widely used in the study of the three-body charmless hadronic  $B$  meson decays [35–41]. In Refs. [40, 41], the authors studied the nonresonant contributions using heavy meson chiral perturbation theory (HMChPT) [42–44] with some modifications and analysed the resonant contributions with the isobar model in terms of the usual Breit-Wigner formalism [45]. The perturbative QCD (PQCD) approach based on the  $k_T$  factorization theorem [46, 47] has also been adopted in Refs. [48–52].

As discussed in Refs. [46–49], the hard  $b$ -quark decay kernels containing two virtual gluons at leading order is not important due to the power-suppression. The contributions from the region, where there is at least one pair of light mesons having an invariant mass below  $O(\Lambda m_B)$  [46, 47],  $\Lambda = m_B - m_b$  being the  $B$  meson and  $b$  quark mass difference, is dominant. It's reasonable that the dynamics associated with the pair of mesons can be factorized into a two-meson distribution amplitude  $\Phi_{h_1 h_2}$  [53]. As a result, one can describe the typical PQCD factorization formula for a  $B \rightarrow h_1 h_2 h_3$  decay amplitude as the form of [46, 47]

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}. \quad (1)$$

With the hard kernel  $H$  describes the dynamics of the strong and electroweak interactions in three-body hadronic decays in a similar way as the one for the two-body  $B \rightarrow h_1 h_2$  decays, the  $\Phi_B$  and  $\Phi_{h_3}$  are the wave functions for the  $B$  meson and the final state  $h_3$ , which absorb the non-perturbative dynamics in the process. The  $\Phi_{h_1 h_2}$  is the two-hadron ( $h_1$  and  $h_2$ ) distribution amplitude proposed in Refs. [53–59], which describes the structure of the final state  $h_1$ - $h_2$  pair.

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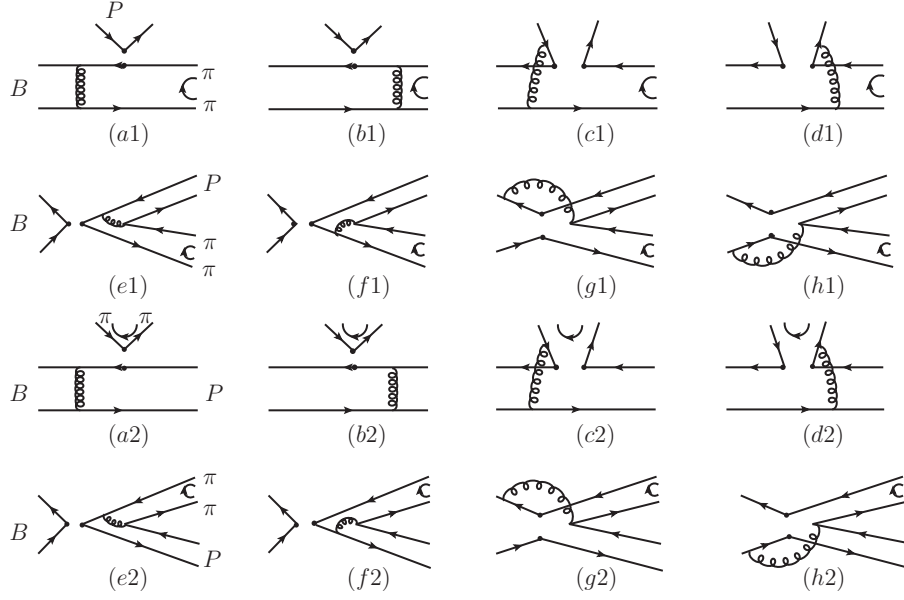


FIG. 1: Typical Feynman diagrams for the quasi-two-body decays  $B \rightarrow P(\rho \rightarrow)\pi\pi$ , where  $B$  stands for the  $B^\pm, B^0$  or  $B_s$  meson and  $P$  denotes  $\pi, K, \eta$  or  $\eta'$ . With  $\alpha = a-h$ , the diagrams  $(\alpha 1)$  for the  $B \rightarrow \rho \rightarrow \pi\pi$  transition and  $(\alpha 2)$  for the  $B \rightarrow P$  transition.

With the help of the two-pion distribution amplitudes, quasi-two-body decays  $B \rightarrow K\rho \rightarrow K\pi\pi$ , the subprocesses of the three-body decays  $B \rightarrow K\pi\pi$ , have been studied in the Ref. [50] in the PQCD approach utilizing framework discussed in [46–49]. The consistency between the PQCD predictions and the data supports the usability of the quasi-two-body framework in Ref. [50] for the study of the three-body hadronic  $B$  decays. In this work, we extend the previous studies in Ref. [50] to the quasi-two-body decays  $B \rightarrow P\rho \rightarrow P\pi\pi$ , with the  $P$  standing for the light pseudoscalar mesons,  $P = (\pi, K, \eta$  or  $\eta')$ , as shown in Fig. 1. In literature, many works have been done for the decays of  $B \rightarrow P\rho$  in two-body framework [30, 34, 60–63] and some of the experimental data could be found in [64–67]. From [50], we know that the width of the resonant state  $\rho$  and the interactions between the final states pion pair will show their effects on the branching ratios especially on the direct  $CP$  violations of the quasi-two-body decays. We should not neglect these effects in  $B \rightarrow P\rho$  decays. In order to describe the strong interactions between the  $P$ -wave resonant state  $\rho$  and the final state pion pair, vector current time-like form factor  $F_\pi$  containing final state interactions between pion pair has been employed in Ref. [50]. Guaranteed by the Watson theorem [68], the results from the  $\pi$ - $\pi$  scattering and  $\tau$  decays for the time-like form factor  $F_\pi$  could be borrowed for the study of quasi-two-body  $B$  meson decays. The detailed discussion of  $F_\pi$  could be found in [50] and its references.

This paper is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. The numerical values, some discussions and the conclusions will be given in last two sections.

## II. FRAMEWORK

For the quasi-two-body  $B \rightarrow P(\rho \rightarrow)\pi\pi$  decays, the weak effective Hamiltonian can be specified as [69]:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{uq} \left[ C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu) \right] - V_{tb}^* V_{tq} \left[ \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\} + \text{H.c.}, \quad (2)$$

with  $q = d, s$ , the  $C_i(\mu)$  ( $i = 1, \dots, 10$ ) are the Wilson coefficients and  $O_i$  are the local four-quark operators.

We let the pion pair and the final state  $P$  move along the direction of  $n = (1, 0, 0_T)$  and  $v = (0, 1, 0_T)$  in the light-cone coordinates, respectively. The  $B$  meson momentum  $p_B$ , the total momentum of the pion pair,  $p = p_1 + p_2$ , and the final state  $P$  momentum  $p_3$  are chosen as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad p = \frac{m_B}{\sqrt{2}}(1, \eta, 0_T), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1 - \eta, 0_T), \quad (3)$$

where  $m_B$  is the mass of  $B$  meson, the variable  $\eta$  is defined as  $\eta = \omega^2/m_B^2$ , the invariant mass squared  $\omega^2 = p^2$ . We define  $\zeta = p_1^+/p^+$  as one of the pion pair's momentum fraction, in terms of which the other kinematic variables of the two pions are expressed as

$$p_1^- = (1 - \zeta)\eta \frac{m_B}{\sqrt{2}}, \quad p_2^+ = (1 - \zeta) \frac{m_B}{\sqrt{2}}, \quad p_2^- = \zeta\eta \frac{m_B}{\sqrt{2}}. \quad (4)$$

We employ  $x_B, z, x_3$  to denote the momentum fraction of the positive quark in each meson,  $k_{BT}, k_T, k_{3T}$  stands for the transverse momentum of the positive quark, respectively. The momentum  $k_B$  of the spectator quark in the  $B$  meson, the momentum  $k$  for the resonant state  $\rho$  and  $k_3$  for the final state  $P$  are of the form of

$$k_B = \left(0, x_B \frac{m_B}{\sqrt{2}}, k_{BT}\right), \quad k = \left(\frac{m_B}{\sqrt{2}}z, 0, k_T\right), \quad k_3 = \left(0, (1 - \eta)x_3 \frac{m_B}{\sqrt{2}}, k_{3T}\right), \quad (5)$$

The momentum fractions  $x_B, z$  and  $x_3$  run from zero to unity.

In this work, we use the wave function [70–74]

$$\Phi_B = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5\phi_B(\mathbf{k}_1), \quad (6)$$

for  $B^+, B^0$  and  $B_s^0$  mesons. And we adopt the widely used distribution amplitude [70–74]

$$\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_B^2} - \frac{1}{2}(\omega_B b)^2\right], \quad (7)$$

for them. With the normalization factor  $N_B$  depends on the value of  $\omega_B$  and  $f_B$ , which is defined through the normalization relation  $\int_0^1 dx \phi_B(x, b=0) = f_B/(2\sqrt{6})$ .  $\omega_B = 0.40 \pm 0.04$  GeV and  $\omega_{B_s} = 0.50 \pm 0.05$  GeV [70, 75, 76] will be employed in the following numerical calculations.

For the final state  $P$  ( $\pi, K, \eta$  or  $\eta'$ ), we have the wave functions [71, 72]

$$\Phi_P(P_3, x_3) \equiv \frac{i}{\sqrt{2N_c}}\gamma_5 [\not{p}_3\phi_P^A(x_3) + m_{03}\phi_P^P(x_3) + m_{03}(\not{p}_3 - 1)\phi_P^T(x_3)], \quad (8)$$

where  $m_{03}$  is the corresponding meson chiral mass,  $P_3$  and  $x_3$  are the momentum and the momentum fraction of  $P$ , respectively. The expressions of the relevant distribution amplitudes of pion and kaon mesons are the following [77–82]:

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}}x(1-x)[1 + 0.44C_2^{3/2}(t)], \quad (9)$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}}[1 + 0.43C_2^{1/2}(t)], \quad (10)$$

$$\phi_\pi^T(x) = \frac{f_\pi}{2\sqrt{6}}(1-2x)[1 + 0.55(10x^2 - 10x + 1)], \quad (11)$$

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}}x(1-x)[1 + 0.17C_1^{3/2}(t) + 0.2C_2^{3/2}(t)], \quad (12)$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}}[1 + 0.24C_2^{1/2}(t)], \quad (13)$$

$$\phi_K^T(x) = -\frac{f_K}{2\sqrt{6}}[C_1^{1/2}(t) + 0.35C_3^{1/2}(t)]. \quad (14)$$

The distribution amplitudes  $\phi_{\eta_{q(s)}}^{A,P,T}$  ( $q=u,d$ ) for  $\eta_{q(s)}$  are given as [77–79, 83]:

$$\phi_{\eta_{q(s)}}^A(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}}6x(1-x)\left[1 + a_1^\eta C_1^{3/2}(2x-1) + a_2^\eta C_2^{3/2}(2x-1) + a_4^\eta C_4^{3/2}(2x-1)\right], \quad (15)$$

$$\phi_{\eta_{q(s)}}^P(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}}\left[1 + (30\eta_3 - \frac{5}{2}\rho_{\eta_{q(s)}}^2)C_2^{1/2}(2x-1) - 3[\eta_3\omega_3 + \frac{9}{20}\rho_{\eta_{q(s)}}^2(1 + 6a_2^\eta)]C_4^{1/2}(2x-1)\right], \quad (16)$$

$$\phi_{\eta_{q(s)}}^T(x) = \frac{f_{q(s)}}{2\sqrt{2N_c}}(1-2x)\left[1 + 6(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{\eta_{q(s)}}^2 - \frac{3}{5}\rho_{\eta_{q(s)}}^2 a_2^\eta)(1 - 10x + 10x^2)\right], \quad (17)$$

with the Gegenbauer moments

$$a_1^\eta = 0, \quad a_2^\eta = 0.44, \quad a_4^\eta = 0.25. \quad (18)$$

The parameters  $\rho_{\eta_q} = 2m_q/m_0^q$  with  $m_0^q = 1.07\text{GeV}$  for  $\eta_q$  and  $\rho_{\eta_s} = 2m_s/m_0^s$  with  $m_0^s = 1.92\text{GeV}$  for  $\eta_s$  [84]. The Gegenbauer polynomials  $C_n^\nu(t)$  ( $n = 1, 2, 3, 4$  and  $\nu = 1/2, 3/2$ ) above could be found in Ref. [82].

In this paper, we consider the meson  $\eta, \eta'$  as mixtures from  $\eta_q$  and  $\eta_s$ :

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (19)$$

with

$$\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \eta_s = s\bar{s}, \quad (20)$$

The mixtures among the  $\eta_q, \eta_s$  and a possible glueball [85–88] will be neglected in this work. For the decay constant and the mixing angle  $\phi$ , we have the forms as [89, 90],

$$f_q = (1.07 \pm 0.02)f_\pi, \quad f_s = (1.34 \pm 0.06)f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ, \quad f_\pi = 0.131. \quad (21)$$

The two-pion distribution amplitudes are the same ones as those being used in Ref. [50],

$$\Phi_{\pi\pi}^P = \frac{1}{\sqrt{2N_c}} [\not{p}\Phi_{\nu\nu=-}^{I=1}(z, \zeta, \omega^2) + \omega\Phi_s^{I=1}(z, \zeta, \omega^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{w(2\zeta - 1)}\Phi_{t\nu=+}^{I=1}(z, \zeta, \omega^2)], \quad (22)$$

with

$$\Phi_{\nu\nu=-}^{I=1} = \phi_0 = \frac{3F_\pi(s)}{\sqrt{2N_c}} z(1-z) \left[ 1 + a_{2\rho}^0 \frac{3}{2} (5(1-2z)^2 - 1) \right] P_1(2\zeta - 1), \quad (23)$$

$$\Phi_s^{I=1} = \phi_s = \frac{3F_s(s)}{2\sqrt{2N_c}} (1-2z) [1 + a_{2\rho}^s (10z^2 - 10z + 1)] P_1(2\zeta - 1), \quad (24)$$

$$\Phi_{t\nu=+}^{I=1} = \phi_t = \frac{3F_t(s)}{2\sqrt{2N_c}} (1-2z)^2 \left[ 1 + a_{2\rho}^t \frac{3}{2} (5(1-2z)^2 - 1) \right] P_1(2\zeta - 1), \quad (25)$$

where the Legendre polynomial  $P_1(2\zeta - 1) = 2\zeta - 1$ . We adopt the same  $F_\pi(s)$  in this work as that in Ref. [50], the relations  $F_{s,t}(s) \approx (f_\rho^T/f_\rho)F_\pi(s)$  [50] will be used in the following section. We make tiny corrections of the Gegenbauer moments for the two-pion distribution amplitudes comparing with those in Ref. [50]. By referring to all the existing data of  $B \rightarrow P(\rho \rightarrow \pi\pi)$  in Ref. [91], we adjust  $a_{2\rho}^0, a_{2\rho}^s, a_{2\rho}^t$  to cater to the data and we have the new Gegenbauer coefficients  $a_{2\rho}^0 = 0.30, a_{2\rho}^s = 0.70, a_{2\rho}^t = -0.40$ .

### III. NUMERICAL RESULTS AND DISCUSSIONS

The following input parameters (the masses, decay constants and QCD scale are in units of GeV) will be used [91] in numerical calculations,

$$\begin{aligned} \Lambda_{\overline{MS}}^{(f=4)} &= 0.25, \quad m_{B^0} = 5.280, \quad m_{B_s} = 5.367, \quad m_{B^\pm} = 5.279, \\ m_{\pi^\pm} &= 0.140, \quad m_{\pi^0} = 0.135, \quad m_{K^\pm} = 0.494, \quad m_{K^0} = 0.498, \\ m_\eta &= 0.548, \quad m_{\eta'} = 0.958, \quad m_{\rho^0} = 0.775, \quad m_{\rho^\pm} = 0.775, \\ m_b &= 4.8, \quad m_c = 1.275, \quad m_s = 0.095, \\ f_B &= 0.19 \pm 0.02, \quad f_{B_s} = 0.236 \pm 0.02, \quad \tau_{B^0} = 1.519 \text{ ps}, \\ \tau_{B_s} &= 1.512 \text{ ps}, \quad \tau_{B^\pm} = 1.638 \text{ ps}, \quad f_\rho = 0.216 \pm 0.003, \quad f_\rho^T = 0.184. \end{aligned} \quad (26)$$

The values of the Wolfenstein parameters are the same as given in Ref. [91]:  $A = 0.814_{-0.024}^{+0.023}$ ,  $\lambda = 0.22537 \pm 0.00061$ ,  $\bar{\rho} = 0.117 \pm 0.021$ ,  $\bar{\eta} = 0.353 \pm 0.013$ .

For the decay  $B \rightarrow P(\rho \rightarrow \pi\pi)$ , the differential branching ratio is written as [91],

$$\frac{d\mathcal{B}}{ds} = \tau_B \frac{|\vec{p}_\pi||\vec{p}_\rho|}{32\pi^3 m_B^3} |\mathcal{A}|^2, \quad (27)$$

TABLE I:  $CP$  averaged branching ratios and direct  $CP$ -violating asymmetries of  $B_{(s)} \rightarrow K(\rho \rightarrow)\pi\pi$  decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-boy results		Experiment
$B^+ \rightarrow K^+(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$4.04^{+0.75}_{-0.58}(\omega_B)^{+0.24}_{-0.20}(a_{2\rho}^t)^{+0.27}_{-0.25}(a_{2\rho}^s)^{+0.22}_{-0.21}(a_{2\rho}^0)$		$3.70 \pm 0.50$
	$\mathcal{A}_{CP}(\%)$	$50.7^{+3.8}_{-2.6}(\omega_B)^{+3.3}_{-4.7}(a_{2\rho}^t)^{+0.0}_{-0.7}(a_{2\rho}^s)^{+0.9}_{-1.5}(a_{2\rho}^0)$		$37.0 \pm 10.0$
$B^0 \rightarrow K^+(\rho^- \rightarrow)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$8.17^{+1.93}_{-1.39}(\omega_B)^{+0.36}_{-0.31}(a_{2\rho}^t)^{+0.46}_{-0.51}(a_{2\rho}^s)^{+0.43}_{-0.43}(a_{2\rho}^0)$		$7.00 \pm 0.90$
	$\mathcal{A}_{CP}(\%)$	$39.7^{+2.6}_{-0.6}(\omega_B)^{+5.1}_{-5.4}(a_{2\rho}^t)^{+0.5}_{-0.0}(a_{2\rho}^s)^{+1.0}_{-0.9}(a_{2\rho}^0)$		$20.0 \pm 11.0$
$B_s^0 \rightarrow K^-(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$19.68^{+7.63}_{-5.18}(\omega_{B_s}) \pm 0.01(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s)^{+0.05}_{-0.06}(a_{2\rho}^0)$		—
	$\mathcal{A}_{CP}(\%)$	$21.8^{+3.7}_{-3.4}(\omega_{B_s}) \pm 0.3(a_{2\rho}^t) \pm 0.2(a_{2\rho}^s) \pm 1.2(a_{2\rho}^0)$		—
$B^+ \rightarrow K^0(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$8.13^{+1.82}_{-1.23}(\omega_B) \pm 0.87(a_{2\rho}^t)^{+0.44}_{-0.43}(a_{2\rho}^s)^{+0.36}_{-0.39}(a_{2\rho}^0)$		$8.00 \pm 1.50$
	$\mathcal{A}_{CP}(\%)$	$13.8^{+3.1}_{-2.9}(\omega_B)^{+2.2}_{-1.9}(a_{2\rho}^t)^{+0.2}_{-0.0}(a_{2\rho}^s)^{+0.2}_{-0.3}(a_{2\rho}^0)$		$-12.0 \pm 17.0$
$B^0 \rightarrow K^0(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$4.39^{+1.12}_{-0.81}(\omega_B) \pm 0.38(a_{2\rho}^t)^{+0.21}_{-0.22}(a_{2\rho}^s)^{+0.19}_{-0.16}(a_{2\rho}^0)$		$4.70 \pm 0.60$
	$\mathcal{A}_{CP}(\%)$	$8.1^{+0.1}_{-0.1}(\omega_B)^{+0.8}_{-0.3}(a_{2\rho}^t)^{+0.8}_{-0.6}(a_{2\rho}^s) \pm 0.0(a_{2\rho}^0)$		—
$B_s^0 \rightarrow \bar{K}^0(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.21^{+0.05}_{-0.01}(\omega_{B_s})^{+0.01}_{-0.00}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.03}_{-0.01}(a_{2\rho}^0)$		—
	$\mathcal{A}_{CP}(\%)$	$63.7^{+13.1}_{-15.2}(\omega_{B_s})^{+5.7}_{-7.0}(a_{2\rho}^t)^{+3.1}_{-4.0}(a_{2\rho}^s)^{+1.5}_{-2.0}(a_{2\rho}^0)$		—

where  $\tau_B$  is the mean lifetime of  $B$  meson, and  $s$  is the invariant mass squared  $s = \omega^2 = p^2$ . The kinematic variables  $|\vec{p}_\pi|$  and  $|\vec{p}_P|$  denote the magnitudes of one  $\pi$  meson in the pion pair and  $P$ 's momenta in the center-of-mass frame of the pion pair,

$$|\vec{p}_\pi| = \frac{1}{2}\sqrt{s - 4m_\pi^2}, \quad |\vec{p}_P| = \frac{1}{2}\sqrt{[(m_B^2 - M_3^2)^2 - 2(m_B^2 + M_3^2)s + s^2]}/s. \quad (28)$$

By using the differential branching fraction in Eq. (27) and the decay amplitudes in the Appendix, we calculate and list the  $CP$  averaged branching ratios ( $\mathcal{B}$ ) and direct  $CP$ -violating asymmetries ( $\mathcal{A}_{CP}$ ) for  $B_{(s)} \rightarrow K(\rho \rightarrow \pi\pi)$  in the third column of Table I,  $B_{(s)} \rightarrow \pi(\rho \rightarrow \pi\pi)$  in Table II and  $B_{(s)} \rightarrow \eta^{(\prime)}(\rho \rightarrow \pi\pi)$  in Table III. The first error of these PQCD predictions comes from  $\omega_B = (0.40 \pm 0.04)$  GeV for  $B^+, B^0$  mesons and  $\omega_{B_s} = (0.50 \pm 0.05)$  GeV for  $B_s$  meson, the second error is from  $a_{2\rho}^t = -0.40 \pm 0.10$ , while the other two errors result from  $a_{2\rho}^s = 0.70 \pm 0.20$  and

TABLE II:  $CP$  averaged branching ratios and direct  $CP$ -violating asymmetries of  $B_{(s)} \rightarrow \pi(\rho \rightarrow)\pi\pi$  decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-boy results		Experiment
$B^+ \rightarrow \pi^+(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$8.84^{+1.48}_{-1.24}(\omega_B)^{+0.12}_{-0.13}(a_{2\rho}^t)^{+1.17}_{-1.11}(a_{2\rho}^s)^{+0.25}_{-0.26}(a_{2\rho}^0)$		$8.30 \pm 1.20$
	$\mathcal{A}_{CP}(\%)$	$-27.5^{+2.3}_{-3.1}(\omega_B)^{+0.9}_{-1.0}(a_{2\rho}^t) \pm 1.4(a_{2\rho}^s) \pm 0.9(a_{2\rho}^0)$		$18.0^{+9.0}_{-17.0}$
$B^0 \rightarrow \pi^+(\rho^- \rightarrow)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$7.85^{+2.60}_{-1.82}(\omega_B)^{+1.77}_{-1.58}(a_{2\rho}^t)^{+0.94}_{-0.91}(a_{2\rho}^s)^{+0.26}_{-0.25}(a_{2\rho}^0)$		$23.00 \pm 2.30$
	$\mathcal{A}_{CP}(\%)$	$-31.4^{+3.4}_{-3.3}(\omega_B)^{+3.2}_{-4.0}(a_{2\rho}^t)^{+1.1}_{-1.6}(a_{2\rho}^s)^{+0.9}_{-0.7}(a_{2\rho}^0)$		$-8.0 \pm 8.0$
$B^0 \rightarrow \pi^-(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$18.78^{+6.92}_{-4.80}(\omega_B)^{+0.56}_{-0.55}(a_{2\rho}^t)^{+0.20}_{-0.21}(a_{2\rho}^s) \pm 0.01(a_{2\rho}^0)$		$23.00 \pm 2.30$
	$\mathcal{A}_{CP}(\%)$	$8.2^{+1.9}_{-1.5}(\omega_B) \pm 0.3(a_{2\rho}^t)^{+0.2}_{-0.1}(a_{2\rho}^s)^{+0.6}_{-0.5}(a_{2\rho}^0)$		$13.0 \pm 6.0$
$B_s^0 \rightarrow \pi^+(\rho^- \rightarrow)\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$0.38 \pm 0.05(\omega_{B_s}) \pm 0.01(a_{2\rho}^t)^{+0.00}_{-0.01}(a_{2\rho}^s)^{+0.02}_{-0.03}(a_{2\rho}^0)$		—
	$\mathcal{A}_{CP}(\%)$	$-4.9^{+0.0}_{-1.7}(\omega_{B_s})^{+1.3}_{-4.4}(a_{2\rho}^t)^{+0.0}_{-2.5}(a_{2\rho}^s)^{+0.6}_{-1.5}(a_{2\rho}^0)$		—
$B_s^0 \rightarrow \pi^-(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$0.41 \pm 0.05(\omega_{B_s})^{+0.00}_{-0.02}(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s)^{+0.02}_{-0.03}(a_{2\rho}^0)$		—
	$\mathcal{A}_{CP}(\%)$	$-36.7^{+0.0}_{-2.5}(\omega_{B_s})^{+2.8}_{-5.4}(a_{2\rho}^t)^{+0.1}_{-0.3}(a_{2\rho}^s)^{+0.0}_{-0.3}(a_{2\rho}^0)$		—
$B^+ \rightarrow \pi^0(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$5.53^{+2.65}_{-1.79}(\omega_B)^{+0.76}_{-0.71}(a_{2\rho}^t)^{+0.49}_{-0.47}(a_{2\rho}^s)^{+0.00}_{-0.02}(a_{2\rho}^0)$		$10.90 \pm 1.40$
	$\mathcal{A}_{CP}(\%)$	$34.9^{+7.3}_{-6.9}(\omega_B)^{+1.6}_{-2.1}(a_{2\rho}^t)^{+1.6}_{-1.7}(a_{2\rho}^s)^{+1.9}_{-1.8}(a_{2\rho}^0)$		$2.0 \pm 11.0$
$B^0 \rightarrow \pi^0(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.11^{+0.06}_{-0.03}(\omega_B)^{+0.02}_{-0.00}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.01}_{-0.00}(a_{2\rho}^0)$		$2.00 \pm 0.50$
	$\mathcal{A}_{CP}(\%)$	$-14.2^{+17.1}_{-4.3}(\omega_B)^{+3.6}_{-0.7}(a_{2\rho}^t)^{+11.3}_{-9.2}(a_{2\rho}^s)^{+2.8}_{-0.0}(a_{2\rho}^0)$		—
$B_s^0 \rightarrow \pi^0(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.35^{+0.06}_{-0.05}(\omega_{B_s}) \pm 0.01(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s) \pm 0.03(a_{2\rho}^0)$		—
	$\mathcal{A}_{CP}(\%)$	$-24.6^{+2.8}_{-0.0}(\omega_{B_s})^{+1.9}_{-0.0}(a_{2\rho}^t)^{+0.0}_{-1.6}(a_{2\rho}^s)^{+0.0}_{-2.6}(a_{2\rho}^0)$		—

TABLE III:  $CP$  averaged branching ratios and direct  $CP$ -violating asymmetries of  $B_{(s)} \rightarrow \eta^{(\prime)}(\rho \rightarrow)\pi\pi$  decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-body results	Experiment
$B^+ \rightarrow \eta(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$6.74^{+2.04}_{-1.50}(\omega_B)^{+0.29}_{-0.27}(a_{2\rho}^t)^{+0.10}_{-0.09}(a_{2\rho}^s)^{+0.02}_{-0.01}(a_{2\rho}^0)$	$7.00 \pm 2.90$
	$\mathcal{ACP}(\%)$	$-0.3^{+0.2}_{-0.0}(\omega_B)^{+0.3}_{-0.2}(a_{2\rho}^t)^{+0.0}_{-0.1}(a_{2\rho}^s)^{+0.0}_{-0.0}(a_{2\rho}^0)$	$11.0 \pm 11.0$
$B^0 \rightarrow \eta(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.03}_{-0.02}(\omega_B)^{+0.03}_{-0.02}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.02}_{-0.00}(a_{2\rho}^0)$	$< 1.5$
	$\mathcal{ACP}(\%)$	$16.3^{+3.3}_{-1.6}(\omega_B)^{+9.1}_{-7.2}(a_{2\rho}^t)^{+0.0}_{-1.9}(a_{2\rho}^s)^{+0.0}_{-1.8}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \eta(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.10^{+0.04}_{-0.02}(\omega_{B_s}) \pm 0.00(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s) \pm 0.00(a_{2\rho}^0)$	—
	$\mathcal{ACP}(\%)$	$19.2^{+0.1}_{-0.2}(\omega_{B_s})^{+0.0}_{-0.4}(a_{2\rho}^t)^{+0.2}_{-0.4}(a_{2\rho}^s)^{+1.5}_{-1.7}(a_{2\rho}^0)$	—
$B^+ \rightarrow \eta'(\rho^+ \rightarrow)\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$4.56^{+1.44}_{-1.02}(\omega_B)^{+0.16}_{-0.13}(a_{2\rho}^t)^{+0.04}_{-0.03}(a_{2\rho}^s)^{+0.02}_{-0.01}(a_{2\rho}^0)$	$9.70 \pm 2.20$
	$\mathcal{ACP}(\%)$	$21.0^{+1.7}_{-1.9}(\omega_B) \pm 1.6(a_{2\rho}^t) \pm 0.2(a_{2\rho}^s) \pm 0.3(a_{2\rho}^0)$	$26.0 \pm 17.0$
$B^0 \rightarrow \eta'(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.05}_{-0.04}(\omega_B)^{+0.01}_{-0.00}(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s) \pm 0.01(a_{2\rho}^0)$	$< 1.3$
	$\mathcal{ACP}(\%)$	$12.8^{+0.0}_{-1.2}(\omega_B)^{+21.1}_{-23.6}(a_{2\rho}^t)^{+8.3}_{-7.3}(a_{2\rho}^s)^{+0.1}_{-0.8}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \eta'(\rho^0 \rightarrow)\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$0.23^{+0.08}_{-0.06}(\omega_{B_s})^{+0.00}_{-0.01}(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s) \pm 0.00(a_{2\rho}^0)$	—
	$\mathcal{ACP}(\%)$	$37.9^{+0.3}_{-0.5}(\omega_{B_s}) \pm 0.2(a_{2\rho}^t) \pm 0.3(a_{2\rho}^s) \pm 0.2(a_{2\rho}^0)$	—

$a_{2\rho}^0 = 0.30 \pm 0.05$ , respectively.

From the numerical results as shown in above three tables, one can address some issues as follows:

- Although we have made small changes for the three Gegenbauer moments  $a_2^{0,s,t}$ , the PQCD predictions for the branching ratios and direct  $CP$  asymmetries of the quasi-two-body decays  $B^+ \rightarrow K^+(\rho^0 \rightarrow)\pi^+\pi^-$ ,  $B^+ \rightarrow K^0(\rho^+ \rightarrow)\pi^+\pi^0$ ,  $B^0 \rightarrow K^+(\rho^- \rightarrow)\pi^-\pi^0$  and  $B^0 \rightarrow K^0(\rho^0 \rightarrow)\pi^+\pi^-$  agree well with those as given previously in Ref. [50]. The PQCD predictions for the decay rates of these four decay modes are consistent with currently available data [91]. For the decay  $B^+ \rightarrow K^+(\rho^0 \rightarrow)\pi^+\pi^-$ , the predicted direct  $CP$  asymmetry  $\mathcal{ACP} = (50.7^{+5.1}_{-5.6})\%$  matches the measured value  $(37.0 \pm 10.0)\%$ .
- For  $B^+ \rightarrow \pi^+(\rho^0 \rightarrow)\pi^+\pi^-$  decay, the PQCD prediction for its branching ratio is well consistent with the world average  $(8.3^{+1.2}_{-1.3}) \times 10^{-6}$  within errors, but its  $CP$  asymmetry is found to be negative:  $\mathcal{ACP} = (-27.5^{+3.0}_{-3.7})\%$  numerically. The BaBar and LHCb measurements for this quantity, however, prefer a positive  $CP$  asymmetry in the  $m(\pi^+\pi^-)$  region peaked at  $m_\rho$ . The theoretical predictions based on the QCDF, PQCD and SCET all give a negative  $CP$  asymmetry of order  $-0.20$  for  $B^+ \rightarrow \rho^0\pi^+$  (see Table XIII of [92]). This puzzle concerning the sign of  $\mathcal{ACP}(\rho^0\pi^+)$  needs to be resolved in the near future.
- The agreements of PQCD predictions with the data could be achieved for  $B \rightarrow \pi(\rho \rightarrow)\pi\pi$  decays comparing with the results in Ref. [60]. The sum of the branching ratios of the  $B^0 \rightarrow \pi^+(\rho^- \rightarrow)\pi^-\pi^0$  and  $B^0 \rightarrow \pi^-(\rho^+ \rightarrow)\pi^+\pi^0$  decays are in consistent with the world average data. The calculated  $\mathcal{ACP}(B^0 \rightarrow \pi^-(\rho^+ \rightarrow)\pi^+\pi^0) = (8.2^{+2.0}_{-1.6})\%$  agree with the data  $(13.0 \pm 6.0)\%$ . We also obtain  $\mathcal{ACP}(B^0 \rightarrow \pi^+(\rho^- \rightarrow)\pi^-\pi^0) = (-31.4^{+4.9}_{-5.5})\%$  which needs to be tested precisely in the future experiments.
- We calculated the branching ratios and  $CP$  violations of the quasi-two-body  $B \rightarrow \eta^{(\prime)}(\rho \rightarrow)\pi\pi$  and find that  $\mathcal{ACP}(B^+ \rightarrow \eta(\rho^+ \rightarrow)\pi^+\pi^0) = (-0.3^{+0.4}_{-0.2})\%$  and  $\mathcal{ACP}(B^+ \rightarrow \eta'(\rho^+ \rightarrow)\pi^+\pi^0) = (21.0^{+2.4}_{-2.5})\%$  agree with the data. The contributions of the tree diagrams are larger than the penguin ones by roughly a factor of 200 for the decay  $B^+ \rightarrow \eta(\rho^+ \rightarrow)\pi^+\pi^0$  and a factor of 40 for the  $B^+ \rightarrow \eta'(\rho^+ \rightarrow)\pi^+\pi^0$ . The tree contribution is therefore dominant for the decay  $B^+ \rightarrow \eta(\rho^+ \rightarrow)\pi^+\pi^0$ . Its direct  $CP$  asymmetry is really small in size. We also give predictions for  $B^0 \rightarrow \eta(\rho^0 \rightarrow)\pi^+\pi^-$  and  $B^0 \rightarrow \eta'(\rho^0 \rightarrow)\pi^+\pi^-$  decays.
- For all the  $B_s \rightarrow K(\pi, \eta^{(\prime)})\rho \rightarrow K(\pi, \eta^{(\prime)})\pi\pi$  decay channels considered in this paper, we can compare our PQCD predictions with those as given in the Table VII and Table VIII of Refs. [82, 93]. From the  $CP$  averaged branching ratios, for example, our results for decays  $B_s \rightarrow K(\pi, \eta^{(\prime)})\rho \rightarrow K(\pi, \eta^{(\prime)})\pi\pi$  are a little larger than the corresponding ones in Table VII of Ref. [82]. As verified in Ref. [50], it may be more appropriate to treat  $B \rightarrow K(\pi, \eta^{(\prime)})\rho$  as the quasi-two-body decays. For  $B_s^0 \rightarrow \pi^-(\rho^+ \rightarrow)\pi^+\pi^0$  and  $B_s^0 \rightarrow \eta(\rho^0 \rightarrow)\pi^+\pi^-$  decays, we obtain sizeable negative  $CP$  asymmetries which could be examined in the forthcoming experiments. Our PQCD predictions for the direct  $CP$  asymmetries of  $B_s^0 \rightarrow K^-(\rho^+ \rightarrow)\pi^+\pi^0$ ,  $B_s^0 \rightarrow \bar{K}^0(\rho^0 \rightarrow)\pi^+\pi^-$ ,  $B_s^0 \rightarrow \eta(\rho^0 \rightarrow)\pi^+\pi^-$  and  $B_s^0 \rightarrow \eta'(\rho^0 \rightarrow)\pi^+\pi^-$  decays are positive and sizable.

## IV. CONCLUSION

In this paper, we calculated the  $CP$ -averaged branching ratios and direct  $CP$ -violating asymmetries of the quasi-two-body decays  $B_{(s)} \rightarrow (\pi, K, \eta, \eta')\rho \rightarrow (\pi, K, \eta, \eta')\pi\pi$  by using the PQCD factorization approach. The two-pion distribution amplitude  $\Phi_{\pi\pi}^P$  with the  $P$ -wave time-like form factor  $F_\pi$  was employed to describe the resonant state  $\rho$  and its interactions with the pion pair. General agreements between the PQCD predictions and the data achieved by making a little adjustments of the Gegenbauer moments of the  $P$ -wave two-pion distribution amplitudes. We listed the PQCD predictions for those considered decay channels, which will be tested at the LHCb and Belle-II experiment.

From the numerical results, we found the following points:

- Except for the  $B \rightarrow \pi^0 \rho^0 \rightarrow \pi^0(\pi^+\pi^-)$  decay mode, the PQCD predictions for the branching ratios of other  $B_{(s)} \rightarrow (\pi, K, \eta, \eta')\rho \rightarrow (\pi, K, \eta, \eta')\pi\pi$  decays agree with currently available data within errors.
- For  $\mathcal{B}(B \rightarrow \pi^0 \rho^0 \rightarrow \pi^0(\pi^+\pi^-))$  decay, the PQCD prediction is about  $(0.11_{-0.03}^{+0.07}) \times 10^{-6}$  and is much smaller than the measured one:  $(2.0 \pm 0.5) \times 10^{-6}$ .
- For  $B^+ \rightarrow \pi^+(\rho^0 \rightarrow \pi^+\pi^-)$  decay mode, we found a negative  $CP$  asymmetry  $(-27.5_{-3.7}^{+3.0})\%$ , which agrees with theoretical predictions based on QCDF or other factorization approaches, but different in sign from the measured ones in the  $m(\pi^+\pi^-)$  region peaked at  $m_\rho$ , as reported by BaBar and LHCb Collaboration. Such difference should be tested in the forthcoming experimental measurements.

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## Appendix A: Decay amplitudes

The total decay amplitude for each considered decay mode in this work are given as follows:

$$\begin{aligned}
 \mathcal{A}(B^+ \rightarrow K^+(\rho^0 \rightarrow \pi^+\pi^-)) = & \frac{G_F}{2} \{ V_{ub}^* V_{us} [(\frac{C_1}{3} + C_2)(F_{e\rho}^{LL} + F_{a\rho}^{LL}) + (C_1 + \frac{C_2}{3})F_{eP}^{LL} + C_2 M_{eP}^{LL} \\
 & + C_1(M_{e\rho}^{LL} + M_{a\rho}^{LL})] - V_{tb}^* V_{ts} [(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10})(F_{e\rho}^{LL} + F_{a\rho}^{LL}) \\
 & + (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8)(F_{e\rho}^{SP} + F_{a\rho}^{SP}) + (C_3 + C_9)(M_{e\rho}^{LL} + M_{a\rho}^{LL}) \\
 & + (C_5 + C_7)(M_{e\rho}^{LR} + M_{a\rho}^{LR}) + \frac{3C_8}{2}M_{eP}^{SP} + \frac{3C_{10}}{2}M_{eP}^{LL} \\
 & + \frac{3}{2}(C_7 + \frac{C_8}{3} + C_9 + \frac{C_{10}}{3})F_{eP}^{LL}] \}, \tag{A1}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}(B^0 \rightarrow K^+(\rho^- \rightarrow \pi^-\pi^0)) = & \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{us} [(\frac{C_1}{3} + C_2)F_{e\rho}^{LL} + C_1 M_{e\rho}^{LL}] - V_{tb}^* V_{ts} [(C_3 + C_9)M_{e\rho}^{LL} \\
 & + (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10})F_{e\rho}^{LL} + (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8)F_{e\rho}^{SP} \\
 & + (C_5 + C_7)M_{e\rho}^{LR} + (\frac{C_3}{3} + C_4 - \frac{1}{2}(\frac{C_9}{3} + C_{10}))F_{a\rho}^{LL} + (C_3 - \frac{C_9}{2})M_{a\rho}^{LL} \\
 & + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8))F_{a\rho}^{SP} + (C_5 - \frac{C_7}{2})M_{a\rho}^{LR}] \}, \tag{A2}
 \end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_s^0 \rightarrow K^-(\rho^+ \rightarrow) \pi^+ \pi^0) &= \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{ud} [(\frac{C_1}{3} + C_2) F_{eP}^{LL} + C_1 M_{eP}^{LL}] - V_{tb}^* V_{td} [(C_3 + C_9) M_{eP}^{LL} \\
&+ (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10}) F_{eP}^{LL} + (C_5 + C_7) M_{eP}^{LR} \\
&+ (\frac{C_3}{3} + C_4 - \frac{1}{2}(\frac{C_9}{3} + C_{10})) F_{aP}^{LL} + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{aP}^{SP} \\
&+ (C_3 - \frac{C_9}{2}) M_{aP}^{LL} + (C_5 - \frac{C_7}{2}) M_{aP}^{LR}] \} , \tag{A3}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow K^0(\rho^+ \rightarrow) \pi^+ \pi^0) &= \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{us} [(\frac{C_1}{3} + C_2) F_{a\rho}^{LL} + C_1 M_{a\rho}^{LL}] - V_{tb}^* V_{ts} [(C_3 - \frac{C_9}{2}) M_{e\rho}^{LL} \\
&+ (\frac{C_3}{3} + C_4 - \frac{1}{2}(\frac{C_9}{3} + C_{10})) F_{e\rho}^{LL} + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{e\rho}^{SP} \\
&+ (C_5 - \frac{C_7}{2}) M_{e\rho}^{LR} + (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10}) F_{a\rho}^{LL} + (C_3 + C_9) M_{a\rho}^{LL} \\
&+ (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8) F_{a\rho}^{SP} + (C_5 + C_7) M_{a\rho}^{LR}] \} , \tag{A4}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^0 \rightarrow K^0(\rho^0 \rightarrow) \pi^+ \pi^-) &= \frac{G_F}{2} \{ V_{ub}^* V_{us} [(C_1 + \frac{C_2}{3}) F_{eP}^{LL} + C_2 M_{eP}^{LL}] - V_{tb}^* V_{ts} [\frac{3C_8}{2} M_{eP}^{SP} \\
&- (\frac{C_3}{3} + C_4 - \frac{1}{2}(\frac{C_9}{3} + C_{10})) (F_{e\rho}^{LL} + F_{a\rho}^{LL}) - (C_3 - \frac{C_9}{2}) (M_{e\rho}^{LL} + M_{a\rho}^{LL}) \\
&- (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) (F_{e\rho}^{SP} + F_{a\rho}^{SP}) - (C_5 - \frac{C_7}{2}) (M_{e\rho}^{LR} + M_{a\rho}^{LR}) \\
&+ \frac{3}{2} (C_7 + \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}) F_{eP}^{LL} + \frac{3C_{10}}{2} M_{eP}^{LL}] \} , \tag{A5}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_s^0 \rightarrow K^0(\rho^0 \rightarrow) \pi^+ \pi^-) &= \frac{G_F}{2} \{ V_{ub}^* V_{ud} [(C_1 + \frac{C_2}{3}) F_{eP}^{LL} + C_2 M_{eP}^{LL}] - V_{tb}^* V_{td} [\frac{3C_8}{2} M_{eP}^{SP} \\
&+ (-\frac{C_3}{3} - C_4 + \frac{5C_9}{3} + C_{10} + \frac{3}{2}(C_7 + \frac{C_8}{3})) F_{eP}^{LL} \\
&+ (-C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2}) M_{eP}^{LL} - (C_5 - \frac{C_7}{2}) (M_{eP}^{LR} + M_{aP}^{LR}) \\
&- (\frac{C_3}{3} + C_4 - \frac{1}{2}(\frac{C_9}{3} + C_{10})) F_{aP}^{LL} - (C_3 - \frac{C_9}{2}) M_{aP}^{LL} \\
&- (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{aP}^{SP}] \} , \tag{A6}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow \pi^+(\rho^0 \rightarrow) \pi^+ \pi^-) &= \frac{G_F}{2} \{ V_{ub}^* V_{ud} [(\frac{C_1}{3} + C_2) (F_{e\rho}^{LL} + F_{a\rho}^{LL} - F_{aP}^{LL}) + (C_1 + \frac{C_2}{3}) F_{eP}^{LL} \\
&+ C_1 (M_{e\rho}^{LL} + M_{a\rho}^{LL} - M_{aP}^{LL}) + C_2 M_{eP}^{LL}] - V_{tb}^* V_{td} [\frac{3C_8}{2} M_{eP}^{SP} \\
&+ (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10}) (F_{e\rho}^{LL} + F_{a\rho}^{LL} - F_{aP}^{LL}) \\
&+ (C_3 + C_9) (M_{e\rho}^{LL} + M_{a\rho}^{LL} - M_{aP}^{LL}) + (-C_5 + \frac{C_7}{2}) M_{eP}^{LR} \\
&+ (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8) (F_{e\rho}^{SP} + F_{a\rho}^{SP} - F_{aP}^{SP}) \\
&+ (C_5 + C_7) (M_{e\rho}^{LR} + M_{a\rho}^{LR} - M_{aP}^{LR}) + (-\frac{C_3}{3} - C_4 + \frac{5}{3} C_9 \\
&+ C_{10} + \frac{3}{2} (C_7 + \frac{C_8}{3})) F_{eP}^{LL} + (-C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2}) M_{eP}^{LL}] \} , \tag{A7}
\end{aligned}$$



$$\begin{aligned}
\mathcal{A}(B^0 \rightarrow \pi^-(\rho^+ \rightarrow) \pi^+ \pi^0) = & \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{ud} [(C_1 + \frac{C_2}{3}) F_{a\rho}^{LL} + (\frac{C_1}{3} + C_2) F_{eP}^{LL} + C_2 M_{a\rho}^{LL} + C_1 M_{eP}^{LL}] \\
& - V_{tb}^* V_{td} [(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10}) F_{eP}^{LL} + (C_4 + C_{10}) M_{a\rho}^{LL} \\
& + (C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} - C_7 - \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}) F_{a\rho}^{LL} \\
& + (C_3 + C_9) M_{eP}^{LL} + (C_5 + C_7) M_{eP}^{LR} + (C_5 - \frac{C_7}{2}) M_{aP}^{LR} \\
& + (\frac{4}{3}(C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}) - C_5 - \frac{C_6}{3} + \frac{1}{2}(C_7 + \frac{C_8}{3})) F_{aP}^{LL} \\
& + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{aP}^{SP} + (C_6 - \frac{C_8}{2}) M_{aP}^{SP} \\
& + (C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}) M_{aP}^{LL} + (C_6 + C_8) M_{a\rho}^{SP} ] \} , \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^0 \rightarrow \pi^+(\rho^- \rightarrow) \pi^- \pi^0) = & \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{ud} [(\frac{C_1}{3} + C_2) F_{e\rho}^{LL} + (C_1 + \frac{C_2}{3}) F_{aP}^{LL} + C_1 M_{e\rho}^{LL} + C_2 M_{aP}^{LL}] \\
& - V_{tb}^* V_{td} [(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10}) F_{e\rho}^{LL} + (C_3 + C_9) M_{e\rho}^{LL} \\
& + (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8) F_{e\rho}^{SP} + (C_5 + C_7) M_{e\rho}^{LR} + (C_6 + C_8) M_{aP}^{SP} \\
& + (\frac{4}{3}(C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}) - C_5 - \frac{C_6}{3} + \frac{1}{2}(C_7 + \frac{C_8}{3})) F_{a\rho}^{LL} \\
& + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{a\rho}^{SP} + (C_3 + C_4 - \frac{C_9}{2} - \frac{C_{10}}{2}) M_{a\rho}^{LL} \\
& + (C_5 - \frac{C_7}{2}) M_{a\rho}^{LR} + (C_6 - \frac{C_8}{2}) M_{a\rho}^{SP} + (C_4 + C_{10}) M_{aP}^{LL} \\
& + (C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} - C_7 - \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}) F_{aP}^{LL} ] \} , \tag{A9}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_s^0 \rightarrow \pi^-(\rho^+ \rightarrow) \pi^+ \pi^0) = & \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{us} [(C_1 + \frac{C_2}{3}) F_{a\rho}^{LL} + C_2 M_{a\rho}^{LL}] - V_{tb}^* V_{ts} [(C_6 + C_8) M_{a\rho}^{SP} \\
& + (C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} - C_7 - \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}) F_{a\rho}^{LL} \\
& + (C_3 + \frac{C_4}{3} - \frac{1}{2}(C_9 + \frac{C_{10}}{3}) - C_5 - \frac{C_6}{3} + \frac{1}{2}(C_7 + \frac{C_8}{3})) F_{aP}^{LL} \\
& + (C_4 - \frac{C_{10}}{2}) M_{aP}^{LL} + (C_6 - \frac{C_8}{2}) M_{aP}^{SP} + (C_4 + C_{10}) M_{a\rho}^{LL} ] \} , \tag{A10}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_s^0 \rightarrow \pi^+(\rho^- \rightarrow) \pi^- \pi^0) = & \frac{G_F}{\sqrt{2}} \{ V_{ub}^* V_{us} [(C_1 + \frac{C_2}{3}) F_{aP}^{LL} + C_2 M_{aP}^{LL}] - V_{tb}^* V_{ts} [(C_4 - \frac{C_{10}}{2}) M_{a\rho}^{LL} \\
& + (C_3 + \frac{C_4}{3} - \frac{1}{2}(C_9 + \frac{C_{10}}{3}) - C_5 - \frac{C_6}{3} + \frac{1}{2}(C_7 + \frac{C_8}{3})) F_{a\rho}^{LL} \\
& + (C_6 - \frac{C_8}{2}) M_{a\rho}^{SP} + (C_4 + C_{10}) M_{aP}^{LL} + (C_6 + C_8) M_{aP}^{SP} \\
& + (C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} - C_7 - \frac{C_8}{3} + C_9 + \frac{C_{10}}{3}) F_{aP}^{LL} ] \} , \tag{A11}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow \pi^0(\rho^+ \rightarrow)\pi^+\pi^0) = & \frac{G_F}{2} \{ V_{ub}^* V_{ud} [(C_1 + \frac{C_2}{3}) F_{e\rho}^{LL} + (\frac{C_1}{3} + C_2)(-F_{a\rho}^{LL} + F_{eP}^{LL} + F_{aP}^{LL}) \\
& + C_2 M_{e\rho}^{LL} + C_1(-M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL})] - V_{tb}^* V_{td} [\frac{3C_8}{2} M_{e\rho}^{SP} \\
& + (-\frac{C_3}{3} - C_4 - \frac{3}{2}(C_7 + \frac{C_8}{3}) + \frac{5C_9}{3} + C_{10}) F_{e\rho}^{LL} \\
& + (-\frac{C_5}{3} - C_6 + \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{e\rho}^{SP} + (-C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2}) M_{e\rho}^{LL} \\
& + (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10})(-F_{a\rho}^{LL} + F_{eP}^{LL} + F_{aP}^{LL}) \\
& + (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8)(-F_{a\rho}^{SP} + F_{aP}^{SP}) + (-C_5 + \frac{C_7}{2}) M_{e\rho}^{LR} \\
& + (C_3 + C_9)(-M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL}) \\
& + (C_5 + C_7)(-M_{a\rho}^{LR} + M_{eP}^{LR} + M_{aP}^{LR})] \} , \tag{A12}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^0 \rightarrow \pi^0(\rho^0 \rightarrow)\pi^+\pi^-) = & -\frac{G_F}{2\sqrt{2}} \{ V_{ub}^* V_{ud} [(C_1 + \frac{C_2}{3})(F_{e\rho}^{LL} - F_{a\rho}^{LL} + F_{eP}^{LL} - F_{aP}^{LL}) \\
& + C_2(M_{e\rho}^{LL} - M_{a\rho}^{LL} + M_{eP}^{LL} - M_{aP}^{LL})] - V_{tb}^* V_{td} [\frac{3C_8}{2}(M_{e\rho}^{SP} + M_{eP}^{SP}) \\
& + (-\frac{C_3}{3} - C_4 - \frac{3}{2}(C_7 + \frac{C_8}{3}) + \frac{5C_9}{3} + C_{10})(F_{e\rho}^{LL} + F_{eP}^{LL}) \\
& + (-\frac{C_5}{3} - C_6 + \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{e\rho}^{SP} + (-C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2})(M_{e\rho}^{LL} + M_{eP}^{LL}) \\
& + (-C_5 + \frac{C_7}{2})(M_{e\rho}^{LR} + M_{eP}^{LR}) - (2C_6 + \frac{C_8}{2})(M_{a\rho}^{SP} + M_{aP}^{SP}) \\
& - (\frac{7C_3}{3} + \frac{5C_4}{3} - 2(C_5 + \frac{C_6}{3}) - \frac{1}{2}(C_7 + \frac{C_8}{3} - \frac{2}{3}(C_9 - C_{10}))) (F_{a\rho}^{LL} + F_{aP}^{LL}) \\
& - (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8))(F_{a\rho}^{SP} + F_{aP}^{SP}) - (C_5 - \frac{C_7}{2})(M_{a\rho}^{LR} + M_{aP}^{LR}) \\
& - (C_3 + 2C_4 - \frac{C_9}{2} + \frac{C_{10}}{2})(M_{a\rho}^{LL} + M_{aP}^{LL})] \} , \tag{A13}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_s^0 \rightarrow \pi^0(\rho^0 \rightarrow)\pi^+\pi^-) = & \frac{G_F}{2\sqrt{2}} \{ V_{ub}^* V_{us} [(C_1 + \frac{C_2}{3})(F_{a\rho}^{LL} + F_{aP}^{LL}) + C_2(M_{a\rho}^{LL} + M_{aP}^{LL})] \\
& - V_{tb}^* V_{ts} [(2(C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3}) - \frac{1}{2}(C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3}))(F_{a\rho}^{LL} + F_{aP}^{LL}) \\
& + (2C_4 + \frac{C_{10}}{2})(M_{a\rho}^{LL} + M_{aP}^{LL}) + (2C_6 + \frac{C_8}{2})(M_{a\rho}^{SP} + M_{aP}^{SP})] \} , \tag{A14}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow \eta_q(\rho^+ \rightarrow)\pi^+\pi^0) = & \frac{G_F}{2} \{ V_{ub}^* V_{ud} [(C_1 + \frac{C_2}{3}) F_{e\rho}^{LL} + (\frac{C_1}{3} + C_2)(F_{a\rho}^{LL} + F_{eP}^{LL} + F_{aP}^{LL}) \\
& + C_2 M_{e\rho}^{LL} + C_1(M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL})] - V_{tb}^* V_{td} [(C_5 - \frac{C_7}{2}) M_{e\rho}^{LR} \\
& + (\frac{7C_3}{3} + \frac{5C_4}{3} - 2(C_5 + \frac{C_6}{3}) - \frac{1}{2}(C_7 + \frac{C_8}{3} - \frac{2}{3}(C_9 - C_{10}))) F_{e\rho}^{LL} \\
& + (\frac{C_5}{3} + C_6 - \frac{1}{2}(\frac{C_7}{3} + C_8)) F_{e\rho}^{SP} + (C_3 + 2C_4 - \frac{C_9}{2} + \frac{C_{10}}{2}) M_{e\rho}^{LL} \\
& + (\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10})(F_{a\rho}^{LL} + F_{eP}^{LL} + F_{aP}^{LL}) \\
& + (\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8)(F_{a\rho}^{SP} + F_{aP}^{SP}) + (C_3 + C_9)(M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL}) \\
& + (C_5 + C_7)(M_{a\rho}^{LR} + M_{eP}^{LR} + M_{aP}^{LR}) + (2C_6 + \frac{C_8}{2}) M_{e\rho}^{SP} \} , \tag{A15}
\end{aligned}$$

$$\begin{aligned} \mathcal{A}(B^+ \rightarrow \eta_s(\rho^+ \rightarrow) \pi^+ \pi^0) &= \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}^* V_{td} \left[ \left( C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} + \frac{1}{2} \left( C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3} \right) \right) F_{e\rho}^{LL} \right. \right. \\ &\quad \left. \left. + (C_4 - \frac{C_{10}}{2}) M_{e\rho}^{LL} + (C_6 - \frac{C_8}{2}) M_{e\rho}^{SP} \right] \right\}, \end{aligned} \quad (\text{A16})$$

$$\mathcal{A}(B^+ \rightarrow \eta(\rho^+ \rightarrow) \pi^+ \pi^0) = \mathcal{A}(B^+ \rightarrow \rho^+ \eta_q) \cos \phi - \mathcal{A}(B^+ \rightarrow \rho^+ \eta_s) \sin \phi, \quad (\text{A17})$$

$$\mathcal{A}(B^+ \rightarrow \eta'(\rho^+ \rightarrow) \pi^+ \pi^0) = \mathcal{A}(B^+ \rightarrow \rho^+ \eta_q) \sin \phi + \mathcal{A}(B^+ \rightarrow \rho^+ \eta_s) \cos \phi, \quad (\text{A18})$$

$$\begin{aligned} \mathcal{A}(B^0 \rightarrow \eta_q(\rho^0 \rightarrow) \pi^+ \pi^-) &= -\frac{G_F}{2\sqrt{2}} \left\{ V_{ub}^* V_{ud} \left[ \left( C_1 + \frac{C_2}{3} \right) (F_{e\rho}^{LL} - F_{a\rho}^{LL} - F_{eP}^{LL} - F_{aP}^{LL}) \right. \right. \\ &\quad + C_2 (M_{e\rho}^{LL} - M_{a\rho}^{LL} - M_{eP}^{LL} - M_{aP}^{LL}) - V_{tb}^* V_{td} \left[ \left( C_5 - \frac{C_7}{2} \right) M_{e\rho}^{LR} \right. \\ &\quad + \left( \frac{7C_3}{3} + \frac{5C_4}{3} - 2(C_5 + \frac{C_6}{3}) - \frac{1}{2} \left( C_7 + \frac{C_8}{3} - \frac{2}{3} (C_9 - C_{10}) \right) \right) F_{e\rho}^{LL} \\ &\quad + \left( \frac{C_5}{3} + C_6 - \frac{1}{2} \left( \frac{C_7}{3} + C_8 \right) \right) F_{e\rho}^{SP} + \left( C_3 + 2C_4 - \frac{C_9}{2} + \frac{C_{10}}{2} \right) M_{e\rho}^{LL} \\ &\quad - \left( -\frac{C_3}{3} - C_4 - \frac{3}{2} \left( C_7 + \frac{C_8}{3} \right) + \frac{5C_9}{3} + C_{10} \right) (F_{a\rho}^{LL} + F_{aP}^{LL}) \\ &\quad - \left( -\frac{C_5}{3} - C_6 + \frac{1}{2} \left( \frac{C_7}{3} + C_8 \right) \right) (F_{a\rho}^{SP} + F_{aP}^{SP}) + \left( 2C_6 + \frac{C_8}{2} \right) M_{e\rho}^{SP} \\ &\quad - \left( -C_3 + \frac{C_9}{2} + \frac{3C_{10}}{2} \right) (M_{a\rho}^{LL} + M_{eP}^{LL} + M_{aP}^{LL}) \\ &\quad - \left( -C_5 + \frac{C_7}{2} \right) (M_{a\rho}^{LR} + M_{eP}^{LR} + M_{aP}^{LR}) - \frac{3C_8}{2} (M_{a\rho}^{SP} + M_{eP}^{SP} + M_{aP}^{SP}) \\ &\quad \left. \left. - \left( -\frac{C_3}{3} - C_4 + \frac{3}{2} \left( C_7 + \frac{C_8}{3} \right) + \frac{5C_9}{3} + C_{10} \right) F_{eP}^{LL} \right] \right\}, \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} \mathcal{A}(B^0 \rightarrow \eta_s(\rho^0 \rightarrow) \pi^+ \pi^-) &= -\frac{G_F}{2} \left\{ -V_{tb}^* V_{td} \left[ \left( C_3 + \frac{C_4}{3} - C_5 - \frac{C_6}{3} + \frac{1}{2} \left( C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3} \right) \right) F_{e\rho}^{LL} \right. \right. \\ &\quad \left. \left. + (C_4 - \frac{C_{10}}{2}) M_{e\rho}^{LL} + (C_6 - \frac{C_8}{2}) M_{e\rho}^{SP} \right] \right\}, \end{aligned} \quad (\text{A20})$$

$$\mathcal{A}(B^0 \rightarrow \eta(\rho^0 \rightarrow) \pi^+ \pi^-) = \mathcal{A}(B^0 \rightarrow \rho^0 \eta_q) \cos \phi - \mathcal{A}(B^0 \rightarrow \rho^0 \eta_s) \sin \phi, \quad (\text{A21})$$

$$\mathcal{A}(B^0 \rightarrow \eta'(\rho^0 \rightarrow) \pi^+ \pi^-) = \mathcal{A}(B^0 \rightarrow \rho^0 \eta_q) \sin \phi + \mathcal{A}(B^0 \rightarrow \rho^0 \eta_s) \cos \phi, \quad (\text{A22})$$

$$\begin{aligned} \mathcal{A}(B_s^0 \rightarrow \eta_q(\rho^0 \rightarrow) \pi^+ \pi^-) &= \frac{G_F}{2\sqrt{2}} \left\{ V_{ub}^* V_{us} \left[ \left( C_1 + \frac{C_2}{3} \right) (F_{a\rho}^{LL} + F_{aP}^{LL}) + C_2 (M_{a\rho}^{LL} + M_{aP}^{LL}) \right. \right. \\ &\quad - V_{tb}^* V_{ts} \left[ -\frac{3}{2} \left( C_7 + \frac{C_8}{3} - C_9 - \frac{C_{10}}{3} \right) (F_{a\rho}^{LL} + F_{aP}^{LL}) \right. \\ &\quad \left. \left. + \frac{3C_8}{2} (M_{a\rho}^{SP} + M_{aP}^{SP}) + \frac{3C_{10}}{2} (M_{a\rho}^{LL} + M_{aP}^{LL}) \right] \right\} \end{aligned} \quad (\text{A23})$$

$$\begin{aligned} \mathcal{A}(B_s^0 \rightarrow \eta_s(\rho^0 \rightarrow) \pi^+ \pi^-) &= \frac{G_F}{2} \left\{ V_{ub}^* V_{us} \left[ \left( C_1 + \frac{C_2}{3} \right) F_{eP}^{LL} + C_2 M_{eP}^{LL} \right] - V_{tb}^* V_{ts} \left[ \frac{3C_8}{2} M_{eP}^{SP} \right. \right. \\ &\quad \left. \left. + \frac{3}{2} \left( C_7 + \frac{C_8}{3} + C_9 + \frac{C_{10}}{3} \right) F_{eP}^{LL} + \frac{3C_{10}}{2} M_{eP}^{LL} \right] \right\}, \end{aligned} \quad (\text{A24})$$

$$\mathcal{A}(B_s^0 \rightarrow \eta(\rho^0 \rightarrow) \pi^+ \pi^-) = \mathcal{A}(B_s^0 \rightarrow \rho^0 \eta_q) \cos \phi - \mathcal{A}(B_s^0 \rightarrow \rho^0 \eta_s) \sin \phi, \quad (\text{A25})$$

$$\mathcal{A}(B_s^0 \rightarrow \eta'(\rho^0 \rightarrow) \pi^+ \pi^-) = \mathcal{A}(B_s^0 \rightarrow \rho^0 \eta_q) \sin \phi + \mathcal{A}(B_s^0 \rightarrow \rho^0 \eta_s) \cos \phi, \quad (\text{A26})$$

where  $G_F$  is the Fermi coupling constant.  $V_{ij}$ 's are the Cabibbo-Kobayashi-Maskawa matrix elements. The functions ( $F_{e\rho}^{LL}, F_{a\rho}^{LL}, M_{e\rho}^{LL}, M_{a\rho}^{LL}, \dots$ ) appeared in above equations are the individual decay amplitudes corresponding to different currents, and their explicit expressions can be found in the Appendix of Ref. [50].

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